Ultrametric Model of Mind, I: Review

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Abstract

We mathematically model Ignacio Matte Blanco's principles of symmetric and asymmetric being through use of an ultrametric topology. We use for this the highly regarded 1975 book of this Chilean psychiatrist and pyschoanalyst (born 1908, died 1995). Such an ultrametric model corresponds to hierarchical clustering in the empirical data, e.g. text. We show how an ultrametric topology can be used as a mathematical model for the structure of the logic that reflects or expresses Matte Blanco's symmetric being, and hence of the reasoning and thought processes involved in conscious reasoning or in reasoning that is lacking, perhaps entirely, in consciousness or awareness of itself. In a companion paper we study how symmetric (in the sense of Matte Blanco's) reasoning can be demarcated in a context of symmetric and asymmetric reasoning provided by narrative text.

1 Introduction

Before introducing and then discussing in detail Matte Blanco's work in section 3, in section 2 we deal with alternative models of mental processes – the geometry or the topology of mental processes. These are neural model centered. Therefore they can be taken as influenced by physical models of the brain. They provide a useful starting point for us because they also avail of ultrametric topological representations and processing frameworks.

It will become clearer later that our primary motivation is not with a physical (and hence, let's assume, observable through physico-chemical and biological processes) view of the brain. Instead our motivation is to have a processing framework to model human reasoning that is, as reasoning, based on data such as text or dialog data, or other behavioral or expressive signals.

In section 3 we survey Matte Blanco's "logico-mathematical", albeit largely descriptive, theory of psychoanalysis.

In section 4 we look at how ultrametrics can provide an appropriate framework for understanding Matte Blanco. In section 5 we tie the ultrametric topology strongly to input data.

In section 6 we note how the unconscious can be vastly more efficient – quicker in reasoning – compared to the conscious mind. An implication of this is why unconscious thought processes are of interest, as well as conscious reasoning, for computation and for decision making.

In general we address through our mathematical – topological and, at times when metric, geometric – modeling how the unconscious differs from the conscious. Modeling for us consists of formulating and defining heuristic data structures, with the intention of allowing us to explore how the unconscious can be expressed in terms of measured data.

2 Topology of Mental Processes: Neuro-Cognitive Approach

Anashin and Khrennikov (2009) present a modeling of cognitive processes and psychology, and point to how it is best to use the ultrametric topology viewpoint rather than the associated p-adic or m-adic (where p is prime, and m is positive integer) algebraic structure. Mental entities are modeled as partially ordered balls (hence sets) in physical space. "Mental topology is ultrametric" (Anashin and Khrennikov, 2009, p. 486). This work pursues that started by Khrennikov at the end of the 1990s and described also in Khrennikov (2004).

A distinction is drawn between the wiring of the brain, a dendritic structure which is, as such, a hierarchical neuronal (possibly non-homogeneous) tree, on the one hand, and on the other hand the "mental trees" of reasoning. Application is proposed to robotics, exemplifying behaviors that encompass certain types of psychical behavior ("evolution of psyche of psycho-robots (and even people interacting with them)", p. 439).

In one chapter (Anashin and Khrennikov, 2009, chapter 14, "m-Adic modeling in cognitive science and psychology"), three models are described for cognitive systems with increasing sophistication as regards psychological and psychotic behaviors (such as neurosis, idée fixe and hysteria, but also emotions). The dynamics of neural networks corresponding to the models are discussed. In chapter 14 of Anashin and Khrennikov (2009), patterns of firing/non-firing neurons are studied, or frequencies of firing. The longest common prefix or Baire metric, which is also an ultrametric, is used. It is noted that these neural computation models, based on p-adic or m-adic dynamics (hence they comprise p-adic or m-adic neural networks) furnish geometrical models of psychology but the claim is not made of relevance for neuro-physiology.

The following chapter in Anashin and Khrennikov (2009) is entitled "Neuronal hierarchy behind the ultrametric mental space". By selecting a priority node in a dendritic graph, rooted trees are determined based on either strings of firing neurons (giving rise to a "mental point") or based on patterns of spik-

ing behavior. Diffusion on these rooted trees leads to establishing probability models on collections of trees. The mental activity in this case is based on a unit given by "the state of a hierarchic neural pathway". Furthermore, "Each psychological function is based on a hierarchic tree of neural pathways", i.e. neuronal trees. Mental encoding of information in this model uses accounting of "frequencies of firings of neurons along the hierarchic neural pathways".

3 Conscious Reasoning and Sub-Conscious Thought Processes: Towards a Cognitive Model that Embraces Both

In the previous section, models of mental processes were used. These models were based on, for example, "mental points" and neural pathways. Applications, as we noted, can be to robotics including robotic behavior – psychotic robotic behavior for example can be understood.

Here in this article, our focus is more restrictive in one sense in that thought processes, and reasoning in particular, are at issue. Rather than robotics as an application for us, instead we are interested in psychoanalysis and in particular text, literary and other bases for empirical data analysis and signal processing.

In other work (e.g. Murtagh et al., 2009) we have taken discussion of quality of artistic creation and studied how well this could be understood computationally, i.e. through pattern recognition. Analogously here, we find in the work of Matte Blanco an excellent basis for the statistical and computational study of human reasoning.

In summary Matte Blanco associated the unconscious with "symmetries" in thought processes; and he associated the conscious with "asymmetries" in reasoning.

3.1 Matte Blanco's Psychoanalysis: A Selective Review

Matte Blanco's *The Unconscious as Infinite Sets* (originally published in 1975; see Matte Blanco, 1998) was, according to the author, "written for psychoanalysts as well as for mathematical philosophers" and is described in Eric Rayner's Foreword as "undoubtedly [his] most fundamental work".

We will begin by summarizing particularly salient aspects of Matte Blanco's work under the points laid out as follows. That will help in showing how the various points can be seen in mathematical terms. Quotations in the following are from Matte Blanco (1998).

- 1. Matte Blanco related his work to Freud's conscious or unconscious.
 - Relative to Freud's work, Matte Blanco had it "largely reformulated in terms of symmetry and asymmetry".

- For him, these were "two kinds or modes of being rather than of existence". The interplay of symmetry and asymmetry is the focus of Matte Blanco's work.
- The upshot of this was that Matte Blanco arrived at what he termed a bi-logical system or bi-logic.

There are "two fundamental types of being which exist within the unity of every man: that of the 'structural' id (or unrepressed unconscious or system unconscious or symmetrical being) which becomes understandable with the help of the principle of symmetry; and that visible in conscious thinking, which can roughly be comprehended in Aristotelian logic."

Freudian consciousness and unconsciousness are reformulated in terms of symmetrical and asymmetrical modes of being. It is to be noted that this is not a Freudian "rational-irrational" polarity but rather, on the side of the symmetric mode of being, the "unrepressed unconscious", or what is "the unconscious by its own nature or structural unconscious". As seen in the development of the theory of Matte Blanco, "It is an attempt at putting in logico-mathematical terms the findings of Freud".

- 2. Symmetrization is a principle which, as shown in Matte Blanco, helps in understanding:
 - schizophrenia, and clinical treatment and practice
 - metaphor and other figures of speech
 - jokes, and disjunctions or abrupt change in discourse
 - emotion and emotionally loaded language
 - dream
 - poetry, literature, art
 - subconscious
 - the "structured" id (or unrepressed unconscious)
 - our system unconscious (or symmetrical being)

What is especially considered by him is "pure symmetry" at the deepest level, i.e. at "the level of being, in contrast to the level of happening" (emphasis in original).

For Matte Blanco, "Symmetrical being is the normal state of man." From it, its counterpart (in a way, its dual), "consciousness or asymmetrical being emerges" and "makes attempts at describing it" (i.e. the primary experience of symmetric being). Justifiably we can consider "symmetrical logic" as the framework of this description by asymmetrical thought of primary symmetrical thought. He says: "the most central trait" of symmetrical being "is the peculiar (extensive) use of symmetrical relations" – hence, "the symmetrical mode of being or symmetrical mode."

- 3. Within a class of things as conceptualized by the thinking person, there is perfect equivalence of class members, implying the following.
 - no contradiction
 - absence of negation
 - displacement
 - space and time vanish
 - no relations of contiguity
 - arising from the last-mentioned: no order
- 4. How a class is defined in practice, or is known to the thinking person, is described in these terms.
 - Because, as elaborated on in Matte Blanco, one class member is in terms of class membership indistinguishable from another class member, we have the following: "the unconscious does not know individuals but only classes or propositional functions which define the class".
 - Further, "The only unity for the (symmetrical) unconscious is the class or set, in which all individuals belonging to it are included. The unconsciousness cannot, therefore, deal with parts, except by treating them as classes or sets."
 - "Consciousness ... when confronted by a whole class can only consider it in two ways: either it focuses on the limits (or definition) of the class, that is, on those precise features which characterize it and distinguish it from all other classes, or it concentrates on the individuals which form the class."
- 5. A class comes about through condensation.
 - "... two impulses which appear incompatible in Aristotelian logic and their union in one expression, ... is accomplished in condensation"
- 6. The principle of generalization relates different classes.
 - We assume various classes.
 - Then "the principle of generalization and the principle of symmetry" are both taken for their explanatory capability in regard to classes.
 - In this way, the "generalizing part [in the human] leads to symbols", since symbols arise out of knowledge of, or awareness of, classes.
 - Classes are structured as, what might be called, "bags of symmetry" (in quotation marks in the original), and also "levels".
- 7. Counterposed to the symmetrical principle in Matte Blanco is the asymmetrical principle.

- It is visible in conscious thinking.
- It can roughly be comprehended in, or expressed through, Aristotelian logic.
- "Asymmetrical being ... perceives reality as divisible or formed by parts and, as such, related to spatio-temporality".
- Symmetrical being can by known only through the glass or prism of asymmetrical being: "Thinking requires asymmetrical relations. So does consciousness."

8. Quantifying the symmetrical.

- "Symmetrical being alone is not observable in man." Even delineating it is "already an asymmetrical ... activity".
- In regard to emotion, the "magnitude of emotion" is understood in terms of "the proportion between symmetrical and asymmetrical thinking".
- "[U]nconscious psychological events are not intrinsically immeasurable" although compared to a physical event being susceptible to just one measurement, instead with unconscious events it is a matter of being susceptible to infinite measurement understood on the basis of the Cantor argument whereby a whole set, being in a bijection with a part of this same set, implies the same countable infinite cardinal for both whole and part sets.
- "By making the individual identical to the class, the principle of symmetry, as seen from an asymmetrical point of view, leads to the infinite set ..."
- "We must ... keep in mind the possibility that if things are viewed in terms of multidimensional space, symmetrical being can actually unfold into an infinite number of asymmetrical relations."
- 9. In free recall, and in other areas besides such as in literature, words are tracers for expressing what lies behind.
 - "Consciousness cannot exist without asymmetrical relations, because the essence of consciousness is to distinguish and to differentiate and that cannot be done with symmetrical relations alone."
 - "Symmetrical being is translated into asymmetrical terms by means of words. Words (i.e. their meanings) are the asymmetrical tools of the translating-unfolding function." (Italics in the original.)
 - We have that "words, abstract things, fulfill the function of differentiating between concepts and also between other things. They are bound to be, therefore, highly asymmetrical in their structure."

• To the foregoing we can add: Text is the "sensory surface" (McKee, 1999, formulated in statistical and computational terms in Murtagh et al., 2009) of the underlying semantics. In section 7 we will return to further motivation as to why words are a good starting point for further analysis and how this can even go towards accessing aspects of underlying symmetrical being.

Thus far, we have selected various central themes from Matte Blanco. This leads us to a conclusion drawn by Lauro-Grotto (2007) that directly follows from Matte Blanco: "... here comes my observation: the structural unconscious, in the way it is reformulated by Matte Blanco, the symmetric mode – all this is homologous to an ultrametric structure. The generalization principle reflects the hierarchical arrangement in which all the stimuli (or concepts) are perceived as belonging to classes, and the classes are clustered into super-classes of increasing generality. Finally, a single omni-comprehensive class is generated."

In a word, an ultrametric topology means that all that we are dealing with is to be found in a hierarchy or a tree structure.

Lauro-Grotto (2007) points to how equi-similar (or equi-distant) stimuli or concepts indicate an ultrametric (or hierarchy, or tree) topology. In this work, we will go further.

We will show how the laying out by Matte Blanco of the symmetric and asymmetric principles leads in a very natural way to an ultrametric topology as a representational model. An ultrametric space is defined by two of the four possible triangle configurations in a Euclidean space, viz. that they be either isosceles with small base, or equilateral.

The isosceles with small base case does not detract one iota from symmetry. Murtagh (2009) explores the many ways and contexts in which a hierarchy expresses symmetry. There is a huge advantage for us in considering especially the isosceles with small base case of ultrametricity: it models novelty, or anomaly, or change. We will illustrate that below (in section 5).

4 Ultrametric Topology, Background and Relevance

Having surveyed Matte Blanco's view of unconscious thought processes expressed as (Matte Blanco's term) symmetry, and conscious reasoning expressed as (again Matte Blanco's term) asymmetry, in this section we will lay out a basis for mathematically modeling these – symmetry, asymmetry – as respectively ultrametric (i.e. metric on a tree or hierarchy) and metric.

As observed by Lauro-Grotto (2007, p. 539), the aspect of anomaly modeling via an ultrametric is nicely consistent with Matte Blanco's symmetrical logic: "... we know that something similar can actually be experienced in finite space when we look at a very distant three-dimensional structure and we perceive it as though it were a single point. Symmetrization of relationships can there-

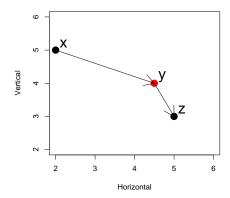


Figure 1: The triangular inequality defines a metric: every triplet of points satisfies the relationship: $d(x, z) \le d(x, y) + d(y, z)$ for distance d.

fore be described as a transition from a metric to an ultrametric conceptual organization."

4.1 Metric

The triangular inequality holds for metrics. An example of a metric is the Euclidean distance, illustrated in Figure 1, where each and every triplet of points satisfies the relationship: $d(x,z) \leq d(x,y) + d(y,z)$ for distance d. Two other relationships also must hold. These are symmetry and positive definiteness, respectively: d(x,y) = d(y,x), and d(x,y) > 0 if $x \neq y$, d(x,y) = 0 if x = y.

Semantic analysis based on a metric embedding is pursued in Murtagh (2005a), especially chapter 5 dealing with many types of textual content, including technical, literary, and philosophy. In Murtagh et al. (2009), in an extensive analysis of film script, it is shown how emotion can be traced out, and this is achieved in an unsupervised way from the text input alone.

4.2 Ultrametric

An ultrametric, compared to a metric, requires a stronger relationship between all triplets of points. The ultrametric is illustrated in Figure 2, left.

To see how an ultrametric is a good mathematical model of anomaly, or exception, or novelty, consider Figure 2, right. Say, for example, there is the situation of seeking best match material, and our search term and the existing material are shown as points in Figure 2, right. If the target population has at least one good match that is close to the query, then this is (let us assume) clearcut. However if all matches in the target population are very unlike the

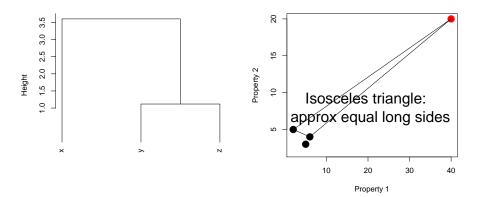


Figure 2: Left: The strong triangular inequality defines an ultrametric: every triplet of points satisfies the relationship: $d(x,z) \leq \max\{d(x,y),d(y,z)\}$ for distance d. Cf. by reading off the hierarchy, how this is verified for all x,y,z. In addition the symmetry and positive definiteness conditions hold for any pair of points. Right: The "new arrival" is on the far right. While we can easily determine the closest target (among the three objects represented by the dots on the lower left), the *approximate* closest target is ambiguous. This motivates the ultrametric as opposed to the metric viewpoint.

query, does it make any sense to choose the closest? Whatever the answer here we are focusing on the inherent ambiguity, which we will note or record in an appropriate way. Figure 2, right, illustrates this situation where the query is the point to the upper right. Relative to the illustration in Figure 2, left, the query point would be associated with terminal node, x.

Note that our illustration in Figure 2, right, uses approximate similarity of the long sides of the triangle shown. This is by way of background for the companion paper, Murtagh (2012).

5 From Data to an Ultrametric: Hierarchical Clustering

5.1 Ultrametric Topology and Hierarchy

The ultrametric topology was introduced by Marc Krasner (1944), the ultrametric inequality having been formulated by Hausdorff in 1934.

Essential motivation for the study of this area is provided by Schikhof (1984, see in particular chapters 18, 19, 20, 21) as follows. Real and complex fields gave rise to the idea of studying any field K with a complete valuation |.| comparable

to the absolute value function. Such fields satisfy the "strong triangle inequality" $|x+y| \leq \max(|x|,|y|)$. Given a valued field, defining a totally ordered Abelian (i.e. commutative) group, an ultrametric space is induced through |x-y| = d(x,y).

Various terms can be used interchangeably for analysis in and over such fields such as p-adic, ultrametric, and non-Archimedean.

The natural geometric ordering of metric valuations is on the real line, whereas in the ultrametric case the natural ordering is a hierarchical tree.

5.2 Further Properties of Ultrametric Spaces

As already noted, in an ultrametric space all triangles are either isosceles with small base, or equilateral. We have here very clear symmetries of shape in an ultrametric topology. These symmetry "patterns" can be used to fingerprint data sets and time series: see Murtagh (2004, 2005b) and Ezhov et al. (2008) for many examples of this.

Some further properties that are studied in Lerman (1981, chapter 0, part IV), Chakraborty (2005) and van Rooij (1978) are as follows.

- 1. Every point of a circle in an ultrametric space is a center of the circle.
- 2. In an ultrametric topology, every ball, i.e. cluster, is both open and closed (termed *clopen*).
- 3. An ultrametric space is 0-dimensional.

It is clear that an ultrametric topology is very different from our intuitive, or Euclidean, notions. The most important point to keep in mind is that in an ultrametric space everything "lives" in a hierarchy expressed by a tree.

The fact that every member of a class can be taken as the center of a circle points to a strong form of equivalence of the class members. Lerman (1981) and Chakraborty (2005) discuss this property.

The property of being both topologically open and closed is due to these properties of a set not being contraries. Rather, a closed set is such that its complement is open. The set can also be open by virtue of being a union of open sets. Open and closed are simultaneous properties in this case. It is important to note that were we to take our cluster members (concepts or whatever) on the real number line, then the context would be different.

The clopen property lends itself well to a class being defined either by what it is not, or being defined through it being a union of other classes.

A 0-dimensional space is studied in van Rooij (1978) or Chakraborty (2005). If the space is constituted of clopen sets – there is a countable base or covering of clopen sets – then it is 0-dimensional.

The dimensionality of a collection of points is 0. So, in the present context and informally expressed, to say we have a 0-dimensional space is the same as saying that each cluster or class we are dealing with will always be akin to a point. That is also to claim that the members of the class, for our purposes here, are tantamount to being identical.

To conclude, therefore, we have sought to show how well an ultrametric space models Matte Blanco's symmetry, as surveyed in section 3.1, and how ultrametric space provides a framework for understanding symmetrical being, or a mathematical model of symmetrical being.

5.3 Inducing an Ultrametric through Agglomerative Hierarchical Clustering

Inducing an ultrametric means, in practice, building a hierarchical clustering from given data.

A mapping of metric to ultrametric is achieved by an agglomerative hierarchical clustering algorithm, a well-established approach that depends on a cluster (compactness, or connectedness, or other) criterion. If each of n observations is taken as a singleton node in the hierarchical tree, n-1 pairwise agglomerations take place, producing a new node in the tree with each agglomeration. There is redefinition of inter-cluster distance (or, less restrictive, dissimilarity) following each agglomeration. Such a tree is 2-way or binary. A p-adic encoding of observations can be read off (Murtagh, 2009, 2010b).

The agglomerative algorithm sketched out here is improved in practice, for computational efficiency reasons. Instead nearest neighbor chains of cluster centers are created, and updated following agglomerations which take place whenever a reciprocal nearest neighbor pair is found. See Murtagh (2005a) and references therein.

One agglomerative clustering criterion is a minimal connectedness one, giving rise to the single link hierarchical clustering method. It can be shown that the resulting ultrametric between any pair of points (or observations) is less than or equal to the starting distance between this pair of points. For this reason it is termed the sub-dominant ultrametric. Rammal et al. (1986) use the sub-dominant ultrametric.

Finally, in this short discussion of hierarchical clustering, to draw yet another link to the work of Matte Blaco, it is noted in Rayner's (1995) review of Matte Blanco that the latter's investigation of "process of thinking ... emphasizes the essential centrality of classificatory activity at all levels of thought, even in the unconscious." This is a useful background consideration for the introduction to empirical and quantitative data analysis in section 7 below.

5.4 Short Review of Hierarchical Clustering Algorithms

Agglomerative hierarchical clustering has been the dominant approach to constructing embedded classification schemes. It is often helpful to distinguish between *method*, involving a compactness criterion and the target structure of a 2-way tree representing the partial order on subsets of the power set; as opposed to an *implementation*, which relates to the detail of the algorithm used.

As with many other multivariate techniques (i.e., input data consists of measures on an object set crossed by an attribute set, so the obects are said to be multivariate), the objects to be classified have numerical measurements on a set of variables or attributes. Hence, the analysis is carried out on the rows of an array or matrix. If we do not have a matrix of numerical values to begin with, then it may be necessary to construct such a matrix from qualitative or symbolic data. The objects, or rows of the matrix, can be viewed as vectors in a multidimensional space (the dimensionality of this space being the number of variables or columns). A geometric framework of this type is not the only one which can be used to formulate clustering algorithms. Suitable alternative forms of storage of a rectangular array of values are not inconsistent with viewing the problem in geometric terms (and in matrix terms – for example, expressing the adjacency relations in a graph).

Motivation for clustering in general, covering hierarchical clustering and applications, includes the following: analysis of data; interactive user interfaces; storage and retrieval; and pattern recognition.

Surveys of clustering with coverage also of hierarchical clustering include Gordon (1981), March (1983), Jain and Dubes (1988), Gordon (1987), Mirkin (1996), Jain et al. (1999), and Xu and Wunsch (2005). Lerman (1981) and Janowitz (2010) present overarching reviews of clustering including through use of lattices that generalize trees. The case for the central role of hierarchical clustering in information retrieval was made by van Rijsbergen (1979) and continued in the work of Willett (e.g. Griffiths et al., 1984) and others. Various mathematical views of hierarchy, all expressing symmetry in one way or another, are explored in Murtagh (2009).

5.5 Ultrametrics and Logic

The usual ultrametric is an ultrametric distance, i.e. for a set I, $d:I\times I\to\mathbb{R}$ (so the ultrametric distance is a real value). The generalized ultrametric is: $d:I\times I\to\Gamma$, where Γ is a partially ordered set. In other words, the generalized ultrametric distance is a set. With this set one can have a value, so the usual and the generalized ultrametrics can amount to more or less the same in practice (by ignoring the set and concentrating on its associated value). After all, in a dendrogram one does have a set associated with each ultrametric distance value (and this is most conveniently the terminals dominated by a given node; but we could have other designs, like some representative subset or other, of these terminals). Most usefully the set, Γ , is defined from the original attributes, which we denote by the set J; whereas the sets of observations read off a dendrogram are subsets of the observation set (which we label with the index set I). So $\Gamma = 2^J$ (and not 2^I).

In the theory of reasoning, a monotonic operator is rigorous application of a succession of conditionals (sometimes called consequence relations). However: "In order to deal with programs of a more general kind (the so-called disjunctive programs) it became necessary to consider multi-valued mappings", supporting non-monotonic reasoning in the way now to be described (Priess-Crampe and

Ribenboim, 1999, pp. 10, 13). The novelty in the work of Priess-Crampe and Ribenboim (1999, 2000) is that these authors use the generalized ultrametric as a multivalued mapping. (A more critical view of the usefulness of the generalized ultrametric perspective is presented by Kroetzsch, 2006).

The generalized ultrametric approach has been motived (Seda and Hitzler, 1998) as follows. "Situations arise ... in computational logic in the presence of negations which force non-monotonicity of the operators involved". To address non-monotonicity of operators, one approach has been to employ metrics in studying some problematic logic programs. These ideas were taken further in examining quasi-metrics, and generalized ultrametrics i.e. ultrametrics which take values in an arbitrary partially ordered set (not just in the non-negative reals). Seda and Hitzler (1998) "consider a natural way of endowing Scott domains [see Davey and Priestley (2002)] with generalized ultrametrics. This step provides a technical tool [for finding fixpoints – hence for analysis] of non-monotonic operators arising out of logic programs and deductive databases and hence to finding models for these."

A further, similar, viewpoint is (Seda and Hitzler, 2010): "Once one introduces negation, which is certainly implied by the term enhanced syntax ... then certain of the important operators are not monotonic (and therefore not continuous), and in consequence the Knaster-Tarski theorem [i.e. for fixed points; see Davey and Priestley, 2002] is no longer applicable to them. Various ways have been proposed to overcome this problem. One such [approach is to use] syntactic conditions on programs ... Another is to consider different operators ... The third main solution is to introduce techniques from topology and analysis to augment arguments based on order ... [latter include:] methods based on metrics ... on quasi-metrics ... and finally ... on ultrametric spaces."

The convergence to fixed points that are based on a generalized ultrametric system is precisely the study of spherically complete systems and expansive automorphisms. See Murtagh (2009) for a short introduction.

6 Norm-Referenced Reasoning and Unconscious Thought Processes, Contrasted with Prototype-Referenced Reasoning and Conscious Reasoning

In Murtagh (2010b) we develop a generative theory of information. Given that algorithmic complexity views the complexity of an object as the work required to generate it, we characterize the work needed to generate an object in an ultrametric space as ultrametric algorithmic information. The approach uses a hierarchy as a "key" to the generative mechanism for an object. It is a norm-referenced approach.

This leads to further support for the hierarchical model, hence an ultrametric topology, as a model for unconscious thought, as we will now discuss.

In Giese and Leopold (2005), it is found that norm-referenced encoding of human faces is a more likely mechanism in facial recognition, compared to example-based encoding. The former is with reference to an average or norm, whereas the latter is relative to prototypical faces.

Leopold et al. (2006) reinforce this: "The main finding was a striking tendency for neurons to show tuning that appeared centered about the average face". They suggest that norm-referencing is helpful for making face recognition robust relative to viewing angle, facial expression, age, and other variable characteristics. Finally they suggest: "Norm-based mechanisms, having adapted to our precise needs in face recognition, may also help explain why our [human] face recognition is so immediate and effortless..."

A wide range of experimental psychology results are presented by Dijksterhuis and Nordgren (2006) to support the link between norm-referenced reasoning and unconscious reasoning, on the one hand, contrasted with the link between prototype-referenced reasoning and conscious thinking, on the other hand. We will pursue some discussion of these links since they provide a most consistent backdrop to our work.

Encoding of information is fundamental. "Thinking about an object implies that the representation of that object in memory changes." Furthermore, "information acquisition" remains crucial for either form of thought, conscious or unconscious.

Dijksterhuis and Nordgren (2006) point to how conscious thought can process between 10 and 60 bits per second. In reading, one processes about 45 bits per second, which corresponds to the time it takes to read a fairly short sentence. However the visual system alone processes about 10 million bits per second. It is concluded from this that the conscious thinking process in humans is very low, compared to the processing capacity of the entire human perception system.

We advance here a hypothesis as to why human thinking includes unconscious as well as conscious thought. Namely, we note that *conscious* reasoning is slow compared to the vastly more efficient and dramatically faster processing speed of *unconscious* thought processes. Dijksterhuis and Nordgren (2006) also point to how unconscious thought is less precise and carries no order, including chronological, information. We have already noted these aspects in Matte Blanco's symmetry and our ultrametric interpretation (subsections 3.1 and 5.2).

Conscious thought therefore is both limited and limiting. A small number of foci of interest ("only one or two attributes") have to take priority. There are inherent limits to conscious thought as a result. As a result of limited capacity, "conscious thought is guided by expectancies and schemas". Limited capacity therefore goes hand in hand with use of stereotypes or schemas. "... people use ... stereotypes (or schemas in general) under circumstances of constrained processing capacity ... [While] this [gives rise to the conclusion] that limited processing capacity during encoding of information leads to more schema use, [current work proposes] that this is also true for thought processes that occur after encoding. ... people stereotype more during impression formation when they think consciously compared to when they think unconsciously. After all,

it is consciousness that suffers from limited capacity."

It may, Dijksterhuis and Nordgren (2006) proceed, be considered counterintuitive that stereotypes are applied in the limited capacity, conscious thought, regime. However stereotypes may be "activated automatically (i.e., unconsciously)", but "they are applied *while we consciously think* about a person or group". Conscious thought is therefore more likely to (unknowingly) attempt "to confirm an expectancy already made".

On the other hand, unconscious thought is less biased in this way, and more slowly integrates information. "Unconscious thought leads to a *better organization* in memory", arrived at through "incubation" of ideas and concepts. "The unconscious works ... aschematically, whereas consciousness works ... schematically". "... conscious thought is more like an architect, whereas unconscious thought behaves more like an archaeologist".

Viewed from the perspective of the work discussed in this subsection, it can be appreciated that our hierarchical and generative description of an object set is a simple model of unconscious thought. That it is simple is clear: to begin with, it is static. Our hierarchical and generative description of an object set (cf. Murtagh, 2010b) is underpinned by the object set being embedded in an ultrametric topology.

We find that, in this framework, the information content is defined from the size of the object set, and not from any given object. To that extent, therefore, the computational (or generative) potential of unconscious thinking is far more powerful that that of conscious thinking.

7 Text Analysis as a Proxy for Both Facets of Bi-Logic

Both conscious or asymmetric reason, and unconscious or symmetric reason, are facets of bi-logic according to Matte Blanco. What he means is that both play a role at different times, that these roles are often complementary, and that the interplay of the two separate domains can be very revealing and instructive.

In this section we address the plausibility of appreciable analysis of content of thought processes based on interrelationships that in turn are frequencies of co-occurrence data. Text will be used as a proxy of underlying thinking, reasoning, conscious phenomena and also, every bit as much, representative of the underlying emotional, dreaming, or other unconscious mental processes. What we are seeking is an approach that is deployable and hence usable in practice.

Words are a means or a medium for getting at the substance and energy of a story, notes McKee (1999, p. 179). Ultimately sets of phrases express such underlying issues (the "subtext", as expressed by McKee) as conflict or emotional connotation. Change and evolution are inherent to a plot. Human emotion is based on particular transitions in thought. So this establishes well the possibility that words and phrases are not only taken literally but can appropriately

capture and represent such transition. Text, says McKee, is the "sensory surface" of a work of art (counterposing it to the subtext, or underlying emotion or perception).

Simple words can express complex underlying reality. Aristotle, for example, used words in common usage to express technically loaded concepts (Murtagh, 2005a, p. 169), and Freud did also.

Rayner (1995) notes the following: "The unconscious largely deals not with particular logically asymmetrically locatable subjects and objects, but with abstract attributes, qualities or notions. Put in another way, these propositional functions are adjectival and adverbial; they lie behind verbal nouns: lovingness, frighteningness and so on." Such words, he notes, are "abstract class attributes, notions or conceptions" and "are the equivalent of the propositional functions of the class".

This has an immediate bearing on the words used in unconscious processes. Rayner (1995) notes the "propositional functions or abstract attributes" or "predicate thinking", that underly the unconscious as fundamental constituents. He also briefly exemplifies this through clinical work in schizophrenia and child abuse by adults.

One could of course deal with other units of thinking, or reasoning, or unconscious processes, other than through words. Chafe (1975), in relating and establishing mappings between memory and story, or narrative, considered the following units.

- 1. *Memory* expressed by a *story* (memory takes the form of an "island"; it is "highly selective"; it is a "disjointed chunk"; but it is not a book, nor a chapter, nor a continuous record, nor a stream).
- 2. Episode, expressed by a paragraph.
- 3. Thought, expressed by a sentence.
- 4. A focus, expressed by a phrase (often these phrases are linguistic "clauses"). Foci are "in a sense, the basic units of memory in that they represent the amount of information to which a person can devote his central attention at any one time".

The "flow of thought and the flow of language" are treated at once, the latter proxying the former, and analyzed in their linear and hierarchical structure as described in other essays in the same volume as Chafe (1979) and in Chafe (1994).

In the companion article to this article, Murtagh (2012), we address the following: Can we attempt to separate out good proxies for symmetrical logic and for asymmetrical logic? To do this, we take a great number of texts, relating to literature, technical writing, and after-the-fact reporting on unconscious thought processes.

8 Conclusion: Matte Blanco's Symmetric Logic as Thought Processes in an Ultrametric Information Space

Ultrametricity, notes Lauro-Grotto (2007), can "be used as a means to generate mental representations that hold in their inner structure all the contradictory aspects of experience and present a smooth surface allowing a kind of 'easy handling' by mental processes."

Chapter 8 of Khrennikov (1997) deals with p-adic dynamical systems in biology and social science. Section 6 is entitled "The human subconscious as a p-adic dynamical system". The conscious is seen as controlling the "gigantic dynamical system" that is the subconscious. A model of the unconscious is set up, based on ideas, that are hierarchically related. Dynamical systems on p-adic number encodings of hierarchies are discussed, including how disruptive or "manic ideas" can be considered in such a context.

This work on p-adic dynamical systems is taken further in the direction of application to cognitive processing in Khrennikov (2004). (See also Khrennikov, 2007, and Khrennikov, 2010.) The author sets out to develop mathematical models for consciousness and allied or analogous mental processes, along the lines of what Newton, Descartes and others started in physics. Right from the start it is noted that "human thinking (as well as many other information processes) is fundamentally a hierarchical process". An m-adic number encoding is used, expressing hierarchy and encompassing therefore an ultrametric topology. Mental spaces, of ideas, are at issue. A p-adic arithmetic, the author notes, is used as a "mind arithmetric". A particular outcome of this work is the linking, by deployment of the same mathematical approaches, of the mathematical models to quantum physical models.

In chapter 7, dealing with "abstract ultrametric information spaces", Khrennikov (2004) enunciates two conjectures that are compatible with the exploration of the ultrametric topologies of ideas that are studied:

- 1. "Cognitive systems (at least some of them) are able to operate simultaneously on all levels of the infinite cognitive hierarchy."
- 2. "Consciousness is created by this infinite volume of information which is concentrated in a finite domain of physical space."

In a theorem (due to A. Lemin) on isometric embedding of an ultrametric space in a Euclidean space, Khrennikov notes that the theorem "might be interpreted as the evidence of impossibility of 'spatial localization of mind' in brain."

There is a clearly a great deal of compatibility between Matte Blanco's work and the other work that we discuss in this article.

Finally we draw a link with symmetry and hierarchy, both understood in quite general terms. Herbert A. Simon, Nobel Laureate in Economics, believed in hierarchy at the basis of the human and social sciences, as the following

quotation shows: "... my central theme is that complexity frequently takes the form of hierarchy and that hierarchic systems have some common properties independent of their specific content. Hierarchy, I shall argue, is one of the central structural schemes that the architect of complexity uses." (Simon, 1996, p. 184).

References

- [1] V. Anashin and A. Khrennikov, *Applied Algebraic Dynamics*, De Gruyter, 2009
- [2] W.L. Chafe, "The flow of thought and the flow of language", In Syntax and Semantics: Discourse and Syntax, ed. by Talmy Givón, vol. 12, 159–181, Academic Press, 1979.
- [3] W. Chafe, Discourse, Consciousness, and Time: The Flow and Displacement of Conscious Experience in Speaking and Writing, University of Chicago Press, 1994.
- [4] P. Chakraborty, "Looking through newly to the amazing irrationals", arXiv: math.HO/0502049v1, 2 Feb. 2005.
- [5] B.A. Davey and H.A. Priestley, *Introduction to Lattices and Order*. Cambridge University Press, 2nd edition, 2002.
- [6] A. Dijksterhuis and L.F. Nordgren, "A theory of unconscious thought", Perspectives on Psychological Science, 1: 95–109, 2006.
- [7] A.A. Ezhov, A.Yu. Khrennikov and S.S. Terentyeva, "Indications of a possible symmetry and its breaking in a many-agent model obeying quantum statistics", *Physical Review*, E 77, 031126, 2008.
- [8] M.A. Giese and D.A. Leopold, "Physiologically inspired neural model for the encoding of face spaces", *Neurocomputing*, 65/66: 93–101, 2005.
- [9] A.D. Gordon, Classification, Chapman and Hall, London, 1981.
- [10] A.D. Gordon, "A review of hierarchical classification", Journal of the Royal Statistical Society A, 150: 119–137, 1987.
- [11] A. Griffiths, L.A. Robinson and P. Willett, "Hierarchic agglomerative clustering methods for automatic document classification", *Journal of Documentation*, 40: 175–205, 1984.
- [12] A. Khrennikov, Non-Archimedean Analysis, Quantum Paradoxes, Dynamical Systems and Biological Models, Kluwer, 1997.
- [13] A. Khrennikov, Information Dynamics in Cognitive, Psychological, Social and Anomalous Phenomena, Kluwer, 2004.

- [14] A. Yu. Khrennikov, "Toward an adequate mathematical model of mental space: Conscious/Unconscious dynamics on m-adic trees", *Biosystems*, 90 (3): 656–675, 2007.
- [15] A. Yu. Khrennikov, "Modelling of psychological behavior on the basis of ultrametric mental space: Encoding of categories by balls", p-Adic Numbers, Ultrametric Analysis, and Applications, 2 (1): 1–20, 2010.
- [16] M. Krasner, "Nombres semi-réels et espaces ultramétriques", Comptes-Rendus de l'Académie des Sciences, Tome II, vol. 219, 433–435, 1944.
- [17] M. Krötzsch. "Generalized ultrametric spaces in quantitative domain theory", *Theoretical Computer Science*, 368: 30–49, 2006.
- [18] A.K. Jain and R.C. Dubes, *Algorithms For Clustering Data* Prentice-Hall, Englwood Cliffs, 1988.
- [19] A.K. Jain, M.N. Murty and P.J. Flynn, "Data clustering: a review", *ACM Computing Surveys*, 31: 264–323, 1999.
- [20] M.F. Janowitz, Ordinal and Relational Clustering, World Scientific, Singapore, 2010.
- [21] R. Lauro-Grotto, "The unconscious as an ultrametric set", American Imago, 64 (4): 535–543, 2007.
- [22] D.A. Leopold, I.V. Bondar and M.A. Giese, "Norm-based face encoding by single neurons in the monkey inferotemporal cortex", *Nature*, 442: 572–575, 2006.
- [23] I.C. Lerman, Classification et Analyse Ordinale des Données, Dunod, 1981.
- [24] S.T. March, "Techniques for structuring database records", ACM Computing Surveys, 15: 45–79, 1983.
- [25] I. Matte Blanco, *The Unconscious as Infinite Sets: An Essay in Bi-Logic*, With a New Foreword by Eric Rayner, Karnac, London, 1998. (Original version 1975).
- [26] R. McKee, Story Substance, Structure, Style, and The Principles of Screenwriting, Methuen, 1999.
- [27] B. Mirkin, Mathematical Classification and Clustering Kluwer, Dordrecht, 1996.
- [28] F. Murtagh, "On ultrametricity, data coding, and computation", *Journal of Classification*, 21: 167–184, 2004.
- [29] F. Murtagh, Correspondence Analysis and Data Coding with R and Java, Chapman & Hall/CRC, 2005a.

- [30] F. Murtagh, "Identifying the ultrametricity of time series", European Physical Journal B, 43: 573–579, 2005b.
- [31] F. Murtagh, "Symmetry in data mining and analysis: A unifying view based on hierarchy", *Proceedings of the Steklov Institute of Mathematics*, 265 (1): 177–198, 2009.
- [32] F. Murtagh, A. Ganz and S. McKie, "The structure of narrative: the case of film scripts", *Pattern Recognition*, 42: 302–312, 2009.
- [33] F. Murtagh, "The correspondence analysis platform for uncovering deep structure in data and information", Computer Journal, 53: 304–315, 2010a.
- [34] F. Murtagh, "On ultrametric algorithmic information", Computer Journal, 53: 405–416, 2010b.
- [35] F. Murtagh, "Ultrametric model of mind, II: Application to text content analysis", preprint (2012).
- [36] S. Priess-Crampe and P. Ribenboim, "Logic programming and ultrametric spaces", *Rendiconti de Matematica*, *Serie VII*, 19: 155–176, 1999.
- [37] S. Priess-Crampe and P. Ribenboim, "Ultrametric spaces and logic programming", *Journal of Logic Programming*, 42: 59–70, 2000.
- [38] R. Rammal, G. Toulouse and M.A. Virasoro, "Ultrametricity for physicists", *Reviews of Modern Physics*, 58: 765–788, 1986.
- [39] E. Rayner, Unconscious Logic: An Introduction to Matte Blanco's Bi-Logic and Its Uses, Routledge, 1995.
- [40] W.H. Schikhof, *Ultrametric Calculus*, Cambridge University Press, 1984. (Chapters 18, 19, 20, 21.)
- [41] A.K. Seda and P. Hitzler, "Generalized ultrametrics, domains and an application to computational logic", *Irish Mathematical Society Bulletin*, 41: 31–43, 1998.
- [42] A.K. Seda and P. Hitzler, "Generalized distance functions in the theory of computation", *Computer Journal*, 53: 443–464, 2010.
- [43] H.A. Simon, The Sciences of the Artificial, MIT Press, Cambridge, MA, 1996.
- [44] C.J. van Rijsbergen, *Information Retrieval*, Butterworths, London, 1979 (2nd ed.).
- [45] A.C.M. van Rooij, Non-Archimedean Functional Analysis, Dekker, 1978.
- [46] R. Xu and D.C. Wunsch, Clustering, IEEE Computer Society Press, 2008.